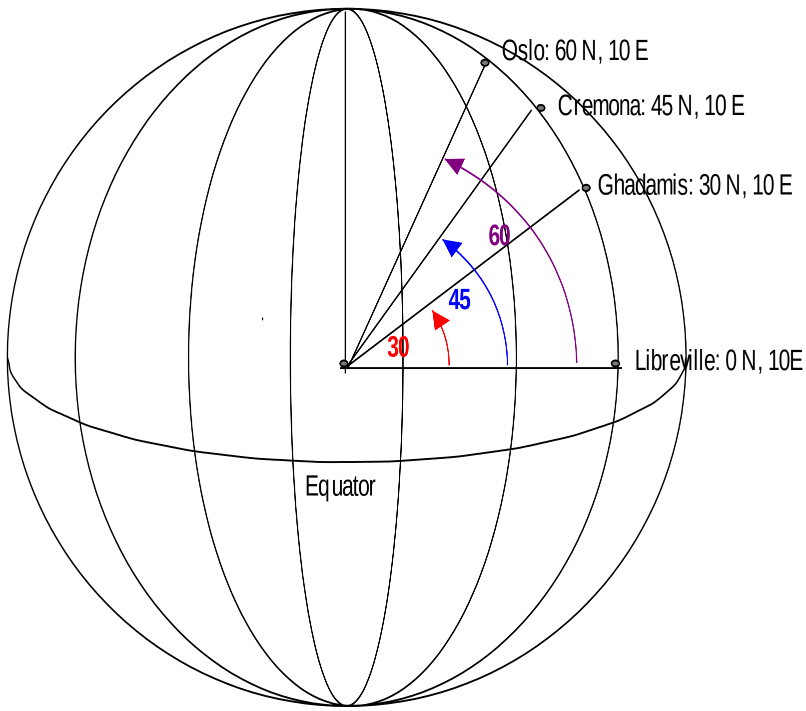
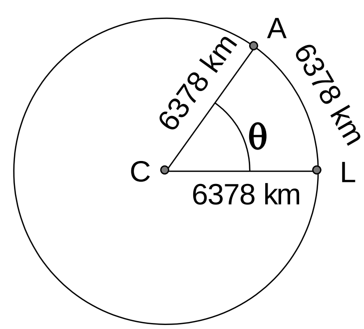
MFH4U – Unit 5: Trigonometric Functions
Lesson 1: Radian Measure

The figure shows the location and approximate coordinates of four cities with longitude 10° E. The cities are Oslo, Norway; Cremona, Italy; Ghadamis, Libya and Libreville, Gabon. The figure shows the central angles formed by connecting these cities with the earth's centre.



1. The radius of the earth is approximately 6378 km. What is the straight-line distance (through the earth!) from Oslo (O) to Libreville (L)?

2. The figure to the right shows the relative positions of Libreville (L) and Alborg, Denmark (A), both at longitude 10°E. The distance between the cities along arc *AL* is about 6378 km. Is the measure (θ) of the central angle ∠ACL greater than 60° or less than 60°. Is Alborg north or south of Oslo? Explain.

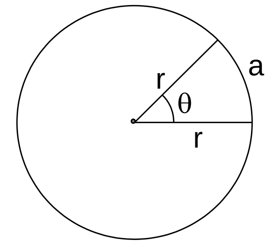


3. Determine the circumference of the earth.

4. Write a proportion involving θ, 360°, and the earth's radius and circumference that you can solve to find θ to the nearest thousandth of a degree.

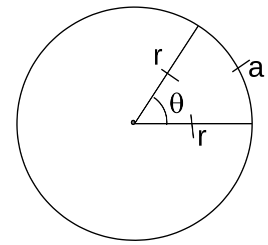
5. If two cities form a central angle with measure 0.5θ, how far apart are they along the surface of the earth? Explain how the value of θ could be a convenient unit of measurement when applying angles to measure distances of the earth.

A convenient way to measure some angles, such as central angles, is radian measure, the ratio of the arc length to the radius of a circle.



On a circle of radius r, the radian measure of a central angle θ that intercepts an arc of length a is given by $\theta = \frac{a}{r}$.

1 radian is defined as the angle subtended by an arc length, a, equal to the radius, r since $\theta = \frac{r}{r} = 1$.



What is the arc length of a 360° angle?
→ Circumference of the circle ∴ $2\pi r$
The corresponding angle, in radians is:

$$\begin{aligned} \theta &= \frac{a}{r} \\ &= \frac{2\pi r}{r} \\ &= 2\pi \end{aligned}$$

360° = 2π radians
or
180° = π radians

∴ 1 radian = $\frac{180^\circ}{\pi}$ and $1^\circ = \frac{\pi}{180}$ radians

Ex. Convert 60° to radians.

$$\begin{aligned} 60^\circ &= 60^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} \\ &= \frac{60\pi}{180} \text{radians} \\ &= \frac{\pi}{3} \text{radians} \end{aligned}$$

Angles measured in radians are normally expressed without any units ∴ 60° = $\frac{\pi}{3}$.

Ex. Convert $\frac{5\pi}{4}$ to degrees.

$$\begin{aligned} \frac{5\pi}{4} &= \frac{5\pi}{4} \cdot \frac{180^\circ}{\pi} \\ &= \frac{5(180^\circ)}{4} \\ &= 225^\circ \end{aligned}$$

Try:

a) Convert 30° to radians.
b) Convert $\frac{7\pi}{6}$ to degrees.

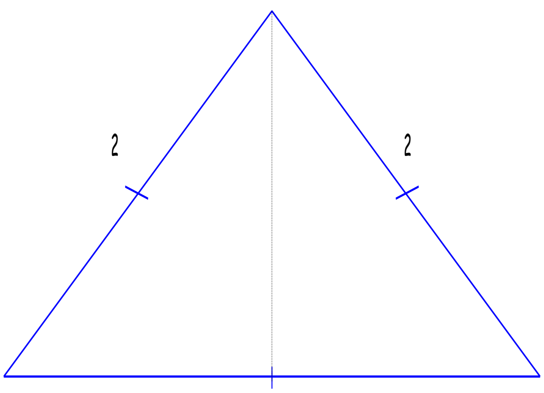
a) $\frac{\pi}{6}$ b) 210°

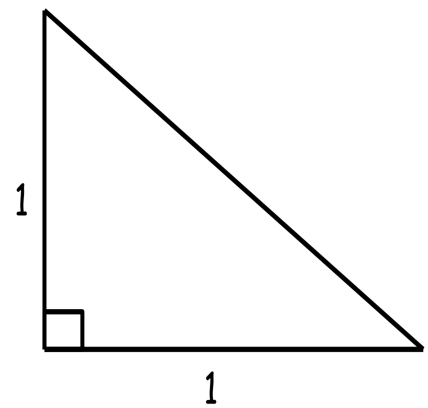
Ex. Two cities are separated by a 1.73 radian central angle. Determine the arc distance between the cities.

$a = \theta r$
$r = 6378$ km

$a = 1.73(6378)$
$a = 11033.94$ km

Recall – special triangles:

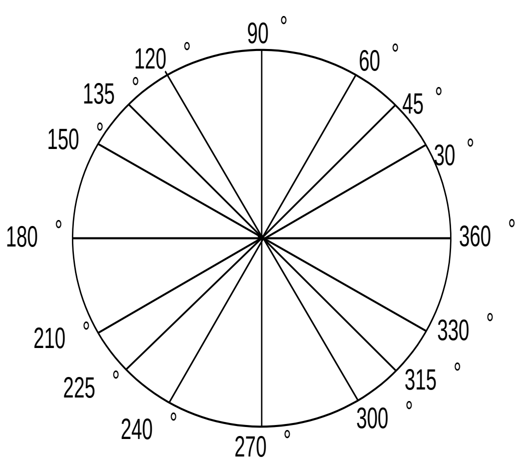




Ex. Determine the exact value of each trigonometric ratio using special triangles then check your answers using a calculator.

a) $\cos\left(\frac{7\pi}{3}\right)$

b) $\cot\left(\frac{7\pi}{4}\right)$



HW: Pg. 321 #2aceg, 3a, 4d, 5, 14
Pg. 330 #5ce, 6ab, 7a, 13

MHF4U – Unit 5

Lesson 2: Graphs of Trigonometric Functions

Explore… Page 333-335

Lesson 3: Transformations of Trigonometric Functions

Recall: Given $y = af\left(k\left(x-d\right)\right)+c$

- ✓ a: vertical stretch/compression and reflection across the x-axis if $a < 0$
- ✓ k: horizontal stretch/compression and reflection across the y-axis if $k < 0$
- ✓ d: horizontal translation
- ✓ c: vertical translation

Given $y = a\sin\left(k\left(\theta-d\right)\right)+c$, we say that a affects the amplitude, k, the period and d, the phase shift.

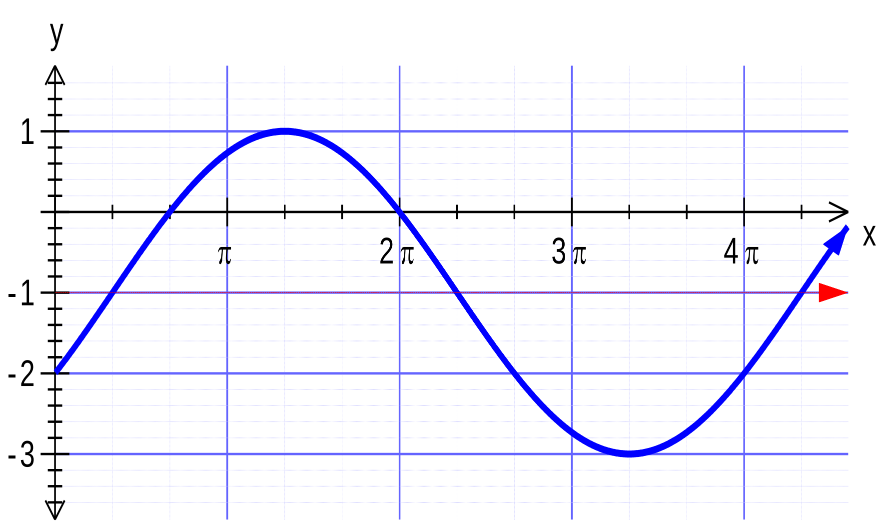
- Changing units to radians will not affect either vertical transformation (amplitude & vertical translation)
- d will now be expressed in radians
- The period of $y=\sin x$ is 2π $\therefore$ the period of a transformed sinusoidal function = $\dfrac{2\pi}{k}$

Ex. Sketch the graph of $y = 2\sin\left(\dfrac{1}{2}\theta - \dfrac{\pi}{6}\right)-1$ over one full period.

Rewrite the equation by factoring out the value of k: $y = 2\sin\left(\dfrac{1}{2}\left(\theta-\dfrac{\pi}{3}\right)\right)-1$

- Amplitude = 2
- Period $= \dfrac{2\pi}{\frac{1}{2}}$

  $= 4\pi$
- Phase shift = $\dfrac{\pi}{3}$ to the right
- Vertical translation: down 1 unit

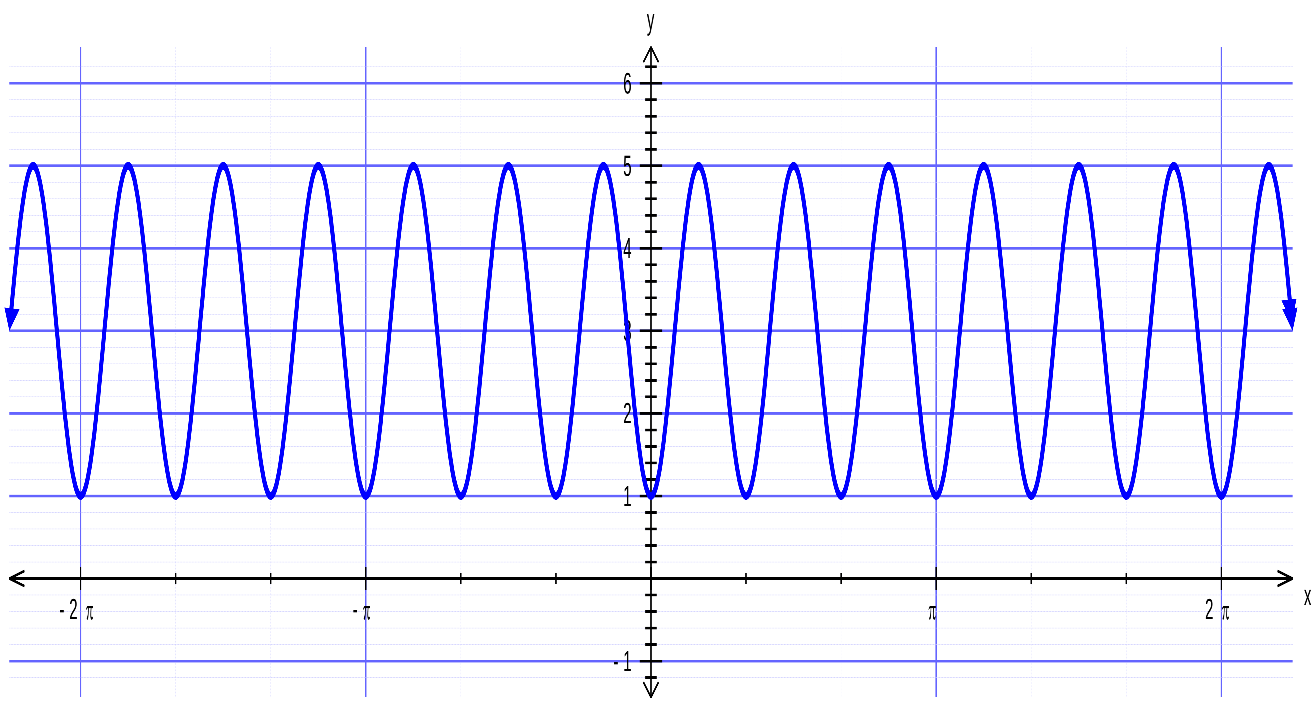


Ex. Write the equation of a sine function with an amplitude of 3 and a period of π.

$$period = \frac{2\pi}{k}$$
$$k = \frac{2\pi}{period}$$
$$k = \frac{2\pi}{\pi}$$
$$k = 2$$

Equation: $y = 3\sin 2\theta$

Ex. Determine two possible cosine equations for the following graph:



- Amplitude = 2
- Period = $\frac{\pi}{3}$ $\therefore$ k = 6
- Phase shift/reflection across x-axis…
- Vertical translation: up 3

Two possible equations are:

$$y = -2\cos(6\theta) + 3$$

or

$$y = 2\cos\left(6\left(\theta - \frac{\pi}{6}\right)\right) + 3$$

HW: Pg. 343 #1d, 4c, 8cf, 10, 14c

MHF4U – Unit 5

Lesson 4: Reciprocal Trigonometric Functions

Recall: Reciprocal trig functions:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Explore… Page 350A - I

Summary

1. Cosecant
   - VA where sin θ = 0
   - Period is 2π
   - Domain: $\{\theta \in \Re \mid \theta \neq n\pi, n \in I\}$
   - Range: $\{y \in \Re \mid |y| > 1\}$

2. Secant
   - VA where cos θ = 0
   - Period is 2π
   - Domain: $\left\{\theta \in \Re \mid \theta \neq (2n-1)\frac{\pi}{2}, n \in I\right\}$
   - Range: $\{y \in \Re \mid |y| > 1\}$

3. Cotangent
   - VA where tan θ = 0
   - Zeros where y = tan θ has VA
   - Period is π
   - Domain: $\{\theta \in \Re \mid \theta \neq n\pi, n \in I\}$
   - Range: $\{y \in \Re\}$

HW: Pg. 353 #2b, 3b, 6, 7

MHF4U – Unit 5
Lesson 5: Modelling Trigonometric Functions

Recall:

Determine the amplitude, period, average y-value, and phase shift of the function $y = 3 + 2\sin\left(2\theta - \frac{\pi}{4}\right)$.

- Amplitude:
- Period:
- Average y-value:
- Phase shift:

The table gives some mean daily temperatures during one year for the town of Roadrunner, New Mexico. The days are numbered beginning with day 1 on January 1.

Day	Mean Temperature (°F)	Day	Mean Temperature (°F)	Day	Mean Temperature (°F)	Day	Mean Temperature (°F)
1	27	95	55	200	83	287	57
12	28	104	57	213	81	301	46
21	28	120	62	224	81	314	45
38	31	138	75	239	73	329	38
51	35	155	78	248	74	340	32
66	39	171	85	257	68	351	29
83	47	188	82	272	62	365	28

1. Plot the data on the graphing calculator (file: U5 L5 Temp Data.tns)
2. What are the maximum and minimum mean daily temperatures? On what days do they occur?
3. What is the mean annual temperature in the town of Roadrunner?
4. Model the mean daily temperature as a periodic function. Explain how you found each of the parameters in your function and what they each mean in this situation.
5. Use your function to predict the mean temperature for day 291 and for today.

A little data collection & modelling…

HW: Pg. 360 #1, 5, 6, 9, 11

MHF4U – Unit 5

Lesson 6: Rates of Change in Trigonometric Functions

Determine rates of change in trigonometric functions as you do in all other functions.

Average Rate of Change:

$$m_{\sec} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Estimating the Instantaneous Rate of Change:

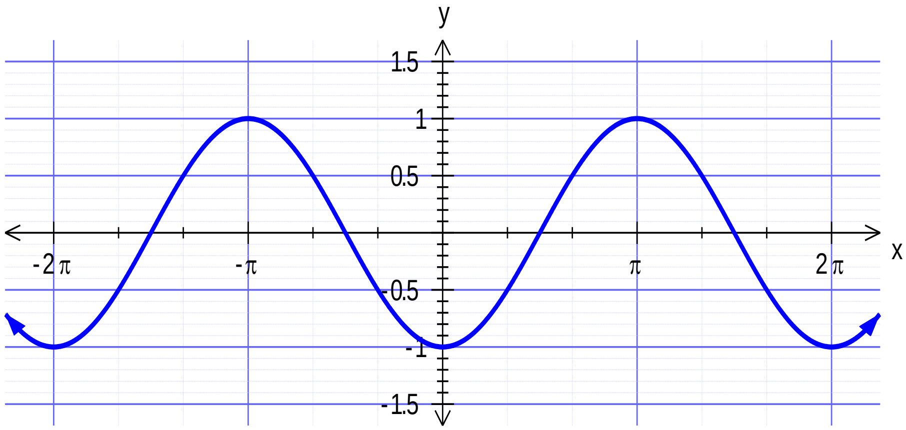
$$m_{\tan} \doteq \frac{f(x+h) - f(x)}{h}, h = 0.001$$

HW: Pg. 370 #2, 4c, 6, 9, 11, 12, 13

MFH4U – Unit 6: Trigonometric Identities and Equations

Lesson 1: Exploring Equivalent Trigonometric Functions

Determine 4 different equations for the following function:



$y = -\cos x$

$y = \cos(x - \pi)$

$y = \cos(x + \pi)$

$y = \sin\left(x - \dfrac{\pi}{2}\right)$

$y = \sin\left(x + \dfrac{3\pi}{2}\right)$

How does the period help us determine equivalent equations?

→ Since the period of sine is 2π, $\sin\theta = \sin(\theta + 2\pi) = \sin(\theta + 4\pi) = \sin(\theta - 2\pi)$. We can generalize this as $\sin\theta = \sin(\theta + 2n\pi)$, $n \in \mathbb{Z}$.

→ Does the same hold true for cosine? Yes: $\cos\theta = \cos(\theta + 2n\pi)$, $n \in \mathbb{Z}$.

→ Does the same hold true for tangent? The period of tangent is π, therefore $\tan\theta = \tan(\theta + n\pi)$, $n \in \mathbb{Z}$.

Explore other ways of creating equivalent expressions: pg. 389 E - N

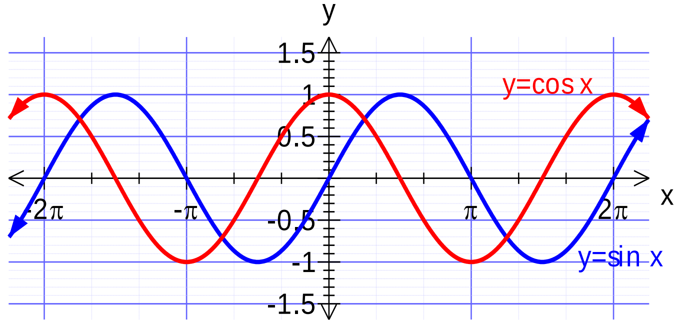
HW: Pg. 392 #1, 2a, 3, 5, 6, 7

MHF4U – Unit 6: Trigonometric Identities and Equations

Lesson 2: Compound Angle Formulas

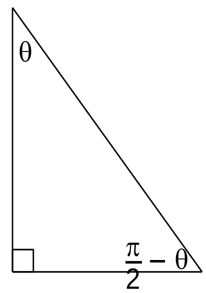
Recall:

1. From sine to cosine and vice-versa:
   - $\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$
   - $\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$

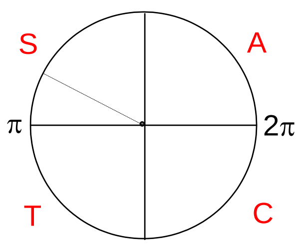
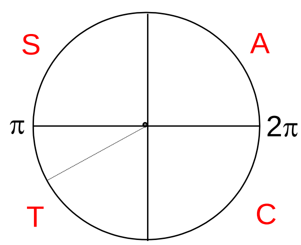
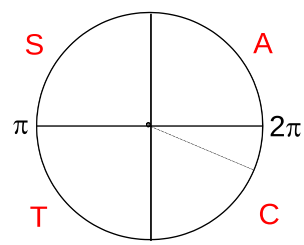


2. Even/odd:
   - $\cos\theta = \cos(-\theta)$ → *even*
   - $\left.\begin{array}{l}\sin(-\theta) = -\sin\theta \\ \tan(-\theta) = -\tan\theta\end{array}\right\}$ *odd*

3. Cofunction Identities:
   - $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$
   - $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$
   - $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

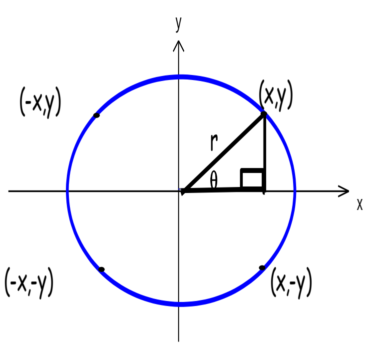


4. Using the related acute angle:

Quadrant II	Quadrant III	Quadrant IV
		
$\sin(\pi-\theta) = \sin\theta$	$\sin(\pi+\theta) = -\sin\theta$	$\sin(2\pi-\theta) = -\sin\theta$
$\cos(\pi-\theta) = -\cos\theta$	$\cos(\pi+\theta) = -\cos\theta$	$\cos(2\pi-\theta) = \cos\theta$
$\tan(\pi-\theta) = -\tan\theta$	$\tan(\pi+\theta) = \tan\theta$	$\tan(2\pi-\theta) = -\tan\theta$

5. $\sin^2\theta + \cos^2\theta = 1$

6.

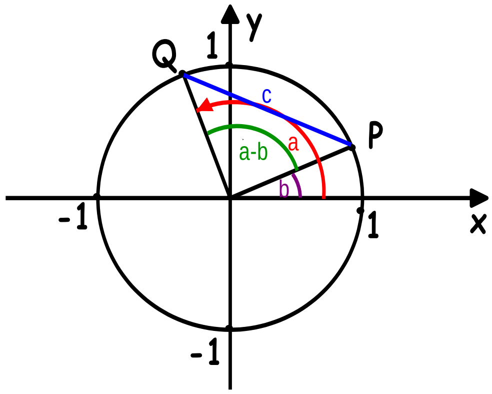


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x},\ x \neq 0$

Compound Angle: angle created by adding or subtracting two or more angles.
Ex: 15° = 45° - 30°



Note: this is the unit circle
Coordinates of P: (cos b, sin b)
Coordinates of Q: (cos a, sin a)

Use the cosine law to find c:

$$c^2 = 1^2 + 1^2 - 2(1)(1)\cos(a-b)$$
$$c^2 = 2 - 2\cos(a-b)$$

Use the distance formula to find c:

$$c = \sqrt{(\sin a - \sin b)^2 + (\cos a - \cos b)^2}$$
$$c^2 = (\sin a - \sin b)^2 + (\cos a - \cos b)^2$$
$$c^2 = \sin^2 a - 2\sin a \cdot \sin b + \sin^2 b + \cos^2 a - 2\cos a \cdot \cos b + \cos^2 b$$
$$c^2 = \sin^2 a + \cos^2 a - 2\sin a \cdot \sin b - 2\cos a \cdot \cos b + \sin^2 b + \cos^2 b$$
$$c^2 = 1 - 2\sin a \cdot \sin b - 2\cos a \cdot \cos b + 1$$
$$c^2 = 2 - 2\sin a \cdot \sin b - 2\cos a \cdot \cos b$$

Equating our two expressions for c^2:

$$2 - 2\cos(a-b) = 2 - 2\sin a \cdot \sin b - 2\cos a \cdot \cos b$$
$$-2\cos(a-b) = -2\sin a \cdot \sin b - 2\cos a \cdot \cos b$$
$$-2\cos(a-b) = -2(\sin a \cdot \sin b + \cos a \cdot \cos b)$$
$$\cos(a-b) = \sin a \cdot \sin b + \cos a \cdot \cos b$$

This is the first of our compound angle formulas.

How can we determine $\cos(a+b)$?

→ Substitute –b for b…

$$\begin{aligned}\cos(a+b) &= \cos(a-(-b)) \\ &= \cos a \cdot \cos(-b) + \sin a \cdot \sin(-b) \\ &= \cos a \cdot \cos b + \sin a \cdot (-\sin b) \\ &= \cos a \cdot \cos b - \sin a \cdot \sin b\end{aligned}$$

To find $\sin(a+b)$ use the cofunction identity: $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$:

$$\begin{aligned}\sin(a+b) &= \cos\left(\frac{\pi}{2}-(a+b)\right)\\ &= \cos\left(\left(\frac{\pi}{2}-a\right)-b\right)\end{aligned}$$

Then use the subtraction formula for cosine.

To find $\tan(a+b)$ use the fact that $\tan\theta = \frac{\sin\theta}{\cos\theta}$:

$$\begin{aligned}\tan(a+b) &= \frac{\sin(a+b)}{\cos(a+b)},\ \cos(a+b)\neq 0\\ &= \frac{\sin a\cos b+\cos a\sin b}{\cos a\cos b-\sin a\sin b}\\ &= \frac{(\sin a\cos b+\cos a\sin b)\left(\frac{1}{\cos a\cos b}\right)}{(\cos a\cos b-\sin a\sin b)\left(\frac{1}{\cos a\cos b}\right)}\\ &= \frac{\frac{\sin a\cos b}{\cos a\cos b}+\frac{\cos a\sin b}{\cos a\cos b}}{\frac{\cos a\cos b}{\cos a\cos b}-\frac{\sin a\sin b}{\cos a\cos b}}\\ &= \frac{\frac{\sin a}{\cos a}+\frac{\sin b}{\cos b}}{1-\frac{\sin a\sin b}{\cos a\cos b}}\\ &= \frac{\tan a+\tan b}{1-\tan a\tan b}\end{aligned}$$

Addition Formulas

$$\boxed{\begin{aligned}&\sin(a+b) = \sin a\cdot\cos b+\cos a\cdot\sin b\\ &\cos(a+b) = \cos a\cdot\cos b-\sin a\cdot\sin b\\ &\tan(a+b) = \frac{\tan a+\tan b}{1-\tan a\cdot\tan b}\end{aligned}}$$

Subtraction Formulas

$$\boxed{\begin{aligned}&\sin(a-b) = \sin a\cdot\cos b-\cos a\cdot\sin b\\ &\cos(a-b) = \cos a\cdot\cos b+\sin a\cdot\sin b\\ &\tan(a-b) = \frac{\tan a-\tan b}{1+\tan a\cdot\tan b}\end{aligned}}$$

Ex. Show that the formula $\cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y$ is true for $x=\frac{\pi}{3}$ and $y=\frac{\pi}{6}$.

$LS=\cos(x-y)$ $=\cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$ $=\cos\left(\frac{2\pi}{6}-\frac{\pi}{6}\right)$ $=\cos\left(\frac{\pi}{6}\right)$ $=\frac{\sqrt{3}}{2}$	$RS=\cos x\cdot\cos y+\sin x\cdot\sin y$ $=\cos\frac{\pi}{3}\cdot\cos\frac{\pi}{6}+\sin\frac{\pi}{3}\cdot\sin\frac{\pi}{6}$ $=\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$ $=\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}$ $=\frac{2\sqrt{3}}{4}$ $=\frac{\sqrt{3}}{2}$

LS = RS

$\therefore$ The formula is valid for $x=\frac{\pi}{3}$ and $y=\frac{\pi}{6}$

Ex. Use an appropriate compound angle formula to determine an exact value for $\sin\frac{\pi}{12}$.

Start by rewriting the given angle as a sum/difference of known 1st quadrant angles: $\frac{\pi}{6},\frac{\pi}{4}\ \&\ \frac{\pi}{3}$.

$$\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$$

$$\begin{aligned}\sin\frac{\pi}{12}&=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\\&=\sin\frac{\pi}{3}\cdot\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\cdot\sin\frac{\pi}{4}\\&=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)\\&=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\\&=\frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

HW: Pg. 400 #1, 3cdf, 5def, 6a, 8cf, 9af,

MHF4U – Unit 6: Trigonometric Identities and Equations

Lesson 3: Double Angle Formulas

Recall:

Addition Formulas

$$\begin{aligned}\sin(a+b)&=\sin a\cdot\cos b+\cos a\cdot\sin b\\ \cos(a+b)&=\cos a\cdot\cos b-\sin a\cdot\sin b\\ \tan(a+b)&=\frac{\tan a+\tan b}{1-\tan a\cdot\tan b}\end{aligned}$$

Subtraction Formulas

$$\begin{aligned}\sin(a-b)&=\sin a\cdot\cos b-\cos a\cdot\sin b\\ \cos(a-b)&=\cos a\cdot\cos b+\sin a\cdot\sin b\\ \tan(a-b)&=\frac{\tan a-\tan b}{1+\tan a\cdot\tan b}\end{aligned}$$

$$\sin^2\theta+\cos^2\theta=1$$

$$\sin^2\theta=1-\cos^2\theta$$

$$\cos^2\theta=1-\sin^2\theta$$

Double Angle Formulas:

1. Determine $\sin(2a)$

$$\begin{aligned}\sin(2a)&=\sin(a+a)\\&=\sin a\cos a+\cos a\sin a\\&=2\sin a\cos a\end{aligned}$$

2. Determine $\cos(2a)$

$$\begin{aligned}\cos(2a)&=\cos(a+a)\\&=\cos a\cos a-\sin a\sin a\\&=\cos^2 a-\sin^2 a\end{aligned}$$

$$\begin{aligned}\cos(2a)&=\cos^2 a-\sin^2 a\\&=\left(1-\sin^2 a\right)-\sin^2 a\\&=1-2\sin^2 a\end{aligned}$$

$$\begin{aligned}\cos(2a)&=\cos^2 a-\sin^2 a\\&=\cos^2 a-\left(1-\cos^2 a\right)\\&=\cos^2 a-1+\cos^2 a\\&=2\cos^2 a-1\end{aligned}$$

3. Determine $\tan(2a)$

$$\begin{aligned}\tan(2a) &= \tan(a+a)\\ &= \frac{\tan a + \tan a}{1 - \tan a \tan a}\\ &= \frac{2\tan a}{1-\tan^2 a}\end{aligned}$$

Ex. Determine $\sin(2x)$ if $\cos x = \frac{1}{2}$ and $\frac{\pi}{2} < 2x < \pi$.

$$\frac{\pi}{2} < 2x < \pi$$
$$\frac{\pi}{4} < x < \frac{\pi}{2}$$
$\therefore x$ is in Quadrant I

$$\begin{aligned}\cos x &= \frac{1}{2}\\ \sin^2 x + \cos^2 x &= 1\\ \sin^2 x + \left(\frac{1}{2}\right)^2 &= 1\\ \sin^2 x + \frac{1}{4} &= 1\\ \sin^2 x &= \frac{3}{4}\\ \sin x &= \pm\frac{\sqrt{3}}{2}\end{aligned}$$

Since x is in Quadrant I, $\sin x = \frac{\sqrt{3}}{2}$

$$\begin{aligned}\sin 2x &= 2\sin x \cos x\\ &= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

Ex. Determine $\cos 2x$ & $\sin 2x$ if $\frac{\pi}{2} < x < \pi$ and $\tan x = \frac{-4}{3}$.

Since x is in Quadrant II, x = -3, y = 4 $\therefore$ r = 5

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x\\ &= \left(\frac{-3}{5}\right)^2 - \left(\frac{4}{5}\right)^2\\ &= \frac{9}{25} - \frac{16}{25}\\ &= \frac{-7}{25}\end{aligned}$$

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ &= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)\\ &= \frac{-24}{25}\end{aligned}$$

HW: Pg. 407 #1, 7, 8, 10, 11a, 13a

MHF4U – Unit 6: Trigonometric Identities and Equations
Lesson 4: Trigonometric Identities

Equations that yield a true statement no matter what value the variable takes are called **identities**.
Ex. $\sin\theta = \sin(\theta + 2\pi)$

To show that an equation is NOT an identity, you only need one counterexample, i.e. one value of the variable that does not work.

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$
$$\tan^2\theta + 1 = \sec^2\theta$$

You can also use the compound angle formulas and double angle formulas:

Addition Formulas

$$\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$$
$$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

Subtraction Formulas

$$\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$$
$$\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$

Double Angle Formulas

$$\begin{aligned} \sin(2a) &= 2\sin a \cos a \\ \cos(2a) &= \cos^2 a - \sin^2 a \\ &= 1 - 2\sin^2 a \\ &= 2\cos^2 a - 1 \\ \tan(2a) &= \frac{2\tan a}{1-\tan^2 a} \end{aligned}$$

Strategies:

- ⇒ Only simplify one side of the identity at a time, starting with the more complicated side first.
- ⇒ NEVER move anything from one side to the other.
- ⇒ Express $\csc\theta, \sec\theta, \tan\theta$ & $\cot\theta$ in terms of $\sin\theta$ & $\cos\theta$.
- ⇒ Simplify algebraically; this often means writing one side with a common denominator.
- ⇒ Look for opportunities to factor.

Ex. Prove $\frac{1}{\sec x} + \frac{\sin x}{\cot x} = \sec x$

$\begin{aligned} LS &= \frac{1}{\sec x} + \frac{\sin x}{\cot x} \\ &= \cos x + \left(\sin x \div \frac{\cos x}{\sin x}\right) \\ &= \cos x + \left(\sin x \cdot \frac{\sin x}{\cos x}\right) \\ &= \cos x + \frac{\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos x} \\ &= \frac{1}{\cos x} \end{aligned}$	$\begin{aligned} RS &= \sec x \\ &= \frac{1}{\cos x} \end{aligned}$

LS = RS

$$\therefore \frac{1}{\sec x} + \frac{\sin x}{\cot x} = \sec x$$

Ex. Prove $\cos(x+y)\cos(x-y)=\cos^2 x+\cos^2 y-1$

$LS=\cos(x+y)\cos(x-y)$ $=(\cos x\cos y-\sin x\sin y)(\cos x\cos y+\sin x\sin y)$ $=\cos^2 x\cos^2 y-\sin^2 x\sin^2 y$ $=\cos^2 x\cos^2 y-(1-\cos^2 x)(1-\cos^2 y)$ $=\cos^2 x\cos^2 y-(1-\cos^2 x-\cos^2 y+\cos^2 x\cos^2 y)$ $=\cos^2 x\cos^2 y-1+\cos^2 x+\cos^2 y-\cos^2 x\cos^2 y$ $=\cos^2 x+\cos^2 y-1$	$RS=\cos^2 x+\cos^2 y-1$

LS = RS

$\therefore\ \cos(x+y)\cos(x-y)=\cos^2 x+\cos^2 y-1$

HW: Pg. 417 #5b, 6, 7, 9, 10abcd

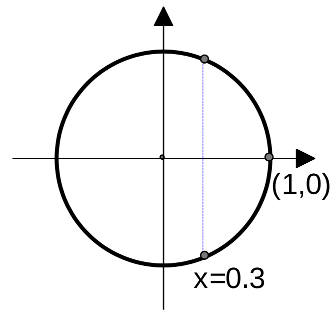
MHF4U – Unit 6

<u>Lesson 5: Solving Linear Trigonometric Equations</u>

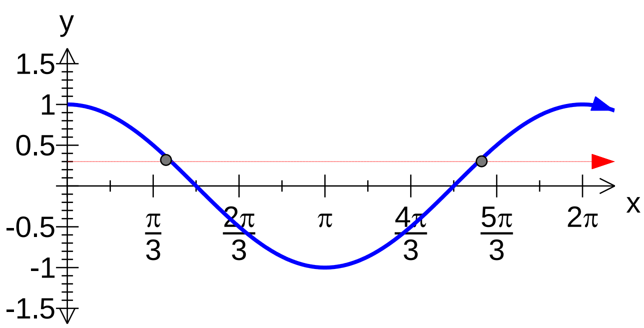
Ex. Solve $\cos x = 0.3$, $0 \le x \le 2\pi$.

The solutions to this equation are angles whose cosine equals 0.3.

If you think of it in terms of the unit circle, we are looking for arc lengths that begin at (1,0) and end where the x-coordinate is 0.3.



If you think of it in terms of the cosine curve, the solutions are where the curve intersects the line $y = 0.3$.

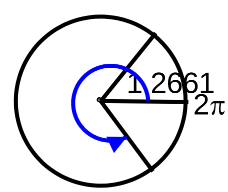


Using your calculator, you can find one solution to this equation:

$$\cos x = 0.3$$
$$x = \cos^{-1}(0.3)$$
$$x \doteq 1.2661$$

Since cosine is positive in quadrants I and IV, the other solution can be found by subtracting the related acute angle from 2π:

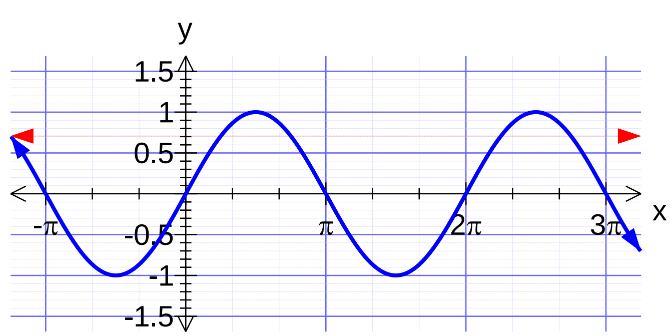
$$x \doteq 2\pi - 1.2661$$
$$\doteq 5.0171$$



Ex. Solve $\sqrt{2}\sin 2\theta = 1$

Rearranging we get:

$$\sqrt{2}\sin 2\theta = 1$$
$$\sin 2\theta = \frac{1}{\sqrt{2}}$$



Using the graph of $y = \sin x$ or the special triangle, we know that

$$\sin x = \frac{1}{\sqrt{2}}$$
$$x = \frac{\pi}{4}$$

$$2\theta = \frac{\pi}{4} + 2n\pi,\ n \in \mathbb{Z}$$
$$\theta = \frac{\pi}{8} + n\pi$$

Since sine is also positive in quadrant III, the second set of values of θ can be found:

$$2\theta = \left(\pi - \frac{\pi}{4}\right) + 2n\pi,\ n \in \mathbb{Z}$$
$$2\theta = \frac{3\pi}{4} + 2n\pi$$
$$\theta = \frac{3\pi}{8} + n\pi$$

If a particular domain was indicated, you would need to substitute values of n in both solutions until you found all the particular solutions in the given domain.

For example, if $0 \le \theta \le 3\pi$:
If $n = 1$,

$$\theta = \frac{\pi}{8} + (1)\pi \qquad \theta = \frac{3\pi}{8} + (1)\pi$$
$$\theta = \frac{9\pi}{8} \qquad \theta = \frac{11\pi}{8}$$

If $n = 2$

$$\theta = \frac{\pi}{8} + (2)\pi \qquad \theta = \frac{3\pi}{8} + (2)\pi$$
$$\theta = \frac{17\pi}{8} \qquad \theta = \frac{19\pi}{8}$$

If $n = 3$, the solutions will be greater than 3π.

Therefore the solution set would be: $\left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{17\pi}{8}, \frac{19\pi}{8}\right\}$

HW: Pg. 426 #1d, 2f, 5, 8, 10c, 11

MHF4U – Unit 6

Lesson 6: Solving Quadratic Trigonometric Equations

In order to solve quadratic trig equations, we need to find a way of writing the quadratic expression as a product of linear expressions. Factoring is a common technique.

Ex. Solve $\cos^2\theta - 1 = 0$.

The LS is a difference of squares:

$$\cos^2\theta - 1 = 0$$
$$(\cos\theta + 1)(\cos\theta - 1) = 0$$

$$\begin{aligned}\cos\theta + 1 &= 0\\ \cos\theta &= -1\\ \theta &= \pi + 2n\pi, n \in \mathbb{Z}\end{aligned}$$

or

$$\begin{aligned}\cos\theta - 1 &= 0\\ \cos\theta &= 1\\ \theta &= 0 + 2n\pi, n \in \mathbb{Z}\\ \theta &= 2n\pi, n \in \mathbb{Z}\end{aligned}$$

Ex. Solve $4\sin^2 x - 3\sin x = 1$.

Rearrange the equation so that one side is equal to zero then factor the trinomial:

$$4\sin^2 x - 3\sin x = 1$$
$$4\sin^2 x - 3\sin x - 1 = 0$$
$$(4\sin x + 1)(\sin x - 1) = 0$$

$$\begin{aligned}4\sin x + 1 &= 0\\ 4\sin x &= -1\\ \sin x &= \frac{-1}{4}\end{aligned}$$

or

$$\begin{aligned}\sin x &= 1\\ x &= \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}\end{aligned}$$

Related acute angle:

$$\theta = \sin^{-1}\left(\frac{1}{4}\right)$$
$$\theta \doteq 0.2527$$

Sine is negative in Quad III & IV:

$$x \doteq (\pi + 0.2527) + 2n\pi, n \in \mathbb{Z}$$
$$x \doteq 3.3943 + 2n\pi$$

$$x \doteq (2\pi - 0.2527) + 2n\pi, n \in \mathbb{Z}$$
$$x \doteq 6.0305 + 2n\pi$$

Ex. Determine solutions to $\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}=2$ for $-2\pi \le x \le 2\pi$.

Simplify the equation so that it is in terms of one trig ratio:

$$\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}=2$$

$$\frac{\cos^2 x+(1+\sin x)^2}{(1+\sin x)(\cos x)}=2$$

$$\frac{\cos^2 x+1+2\sin x+\sin^2 x}{(1+\sin x)(\cos x)}=2$$

$$\frac{2+2\sin x}{(1+\sin x)(\cos x)}=2$$

$$\frac{2(1+\sin x)}{(1+\sin x)(\cos x)}=2$$

$$\frac{2}{\cos x}=2$$

$$\frac{2}{2}=\cos x$$

$$\cos x=1$$

Solution set: {-2π, 0, 2π}

HW: Pg. 435 #1abcd, 7aef, 8abc